MATH-7A TEST 2 (Unit 3 – Polynomial and Rational Functions and Graphs)
100 points * NAME: Solutions

Note: this is a little longer than your test will be.

Fill in the blanks. (2 points each)

(1) What is the equation of the horizontal asymptote of $f(x) = \frac{3x^2+1}{2x^3-5x}$? $y=0$

(2) True or False: If c is a rational zero of P(x) = $2x^3 - 4x^2 + x - 5$ then it is in the list $\pm\left\{1, 2, \frac{1}{5}, \frac{2}{5}\right\}$ False

(3) Is $f(x) = \frac{(x-2)^{3/2}}{x^4\sqrt{x+7}}$ a rational function? no

not polynomials

(4) The graph of $f(x) = \frac{x^2-4}{x-2}$ has a hole at the point $(2,4)$

$f(x) = \frac{(x-2)(x+2)}{x-2} = x+2;\ x\neq 2$

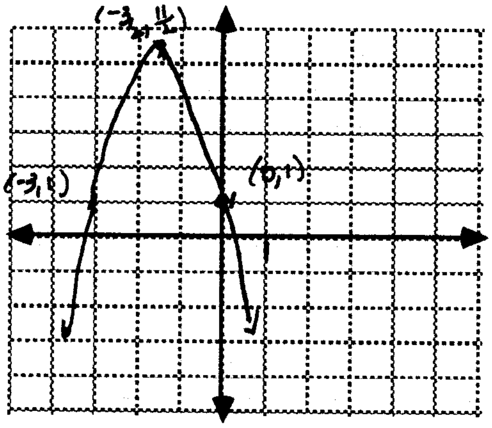
(5) Given $y = \frac{5x}{(x-1)(x+3)}$, as $x \to 1^-$, $y \to$ $-\infty$.

$\approx \frac{5}{-\text{small}\ 4}$

(6) What is the equation of the slant asymptote of $f(x) = \frac{3x^2-4x-5}{x-1}$? $y=3x-1$

$$\begin{array}{r} 3x-1 \\ x-1\,\big)\,3x^2-4x-5 \\ -(3x^2-3x) \\ \hline -x-5 \\ -(-x+1) \\ \hline -6 \end{array}$$

(7) Given the function $f(x) = -2x^2 - 6x + 1$ (6 points)
put f(x) in the form $f(x) = a(x-h)^2 + k$ and sketch the graph. On the graph label the vertex plus one other point. Show scale.



$f(x) = -2(x^2+3x+\frac{9}{4}) + 1 + \frac{9}{2}$

$f(x) = -2(x+\frac{3}{2})^2 + \frac{11}{2}$ Vertex $(-\frac{3}{2}, \frac{11}{2})$

Check: $x = \frac{-b}{2a} = \frac{-(-6)}{2(-2)} = -\frac{3}{2}$

$f(-\frac{3}{2}) = -2\cdot\frac{9}{4} - 6(\frac{-3}{2}) + 1 = \frac{11}{2}$

Check a point in original:

$f(1) = -2-6+1 = -7$

(8) Solve the following. Show appropriate method. Answer in interval notation. (10 points each)

(a) $(x-3)^2(x+2)>0$

$$\begin{array}{ccccccc} - & - & \quad & + & + & \quad & + \\ \hline & & -2 & 0 & & 3 & \end{array}$$

$(-2,3)\cup(3,\infty)$

3 is not included because when $x=3$, $(x-3)^2(x+2)$ equals 0

(b) $\frac{1}{x+1} \leq \frac{4}{x-3}$

Remember: Cannot multiply by $(x+1)(x-3)$ here because we dont know the sign.

$\frac{1}{x+1} - \frac{4}{x-3} \leq 0$

$\frac{x-3-4(x+1)}{(x+1)(x-3)} \leq 0$

$\frac{-3x-7}{(x+1)(x-3)} \leq 0$

numerator $= 0$ $x = -\frac{7}{3}$

denom $= 0$ $x = -1, 3$

$$\begin{array}{cccccccc} + & + & & - & + & + & & - \\ \hline & & -\frac{7}{3} & -1 & 0 & & 3 & \end{array}$$

$[-\frac{7}{3}, -1)\cup(3,\infty)$

1,2 $\qquad$ 1,2,3,4,6,12

(9) Given the polynomial $P(x) = 2x^4 - 5x^3 + 5x^2 - 20x - 12$ (12 points)

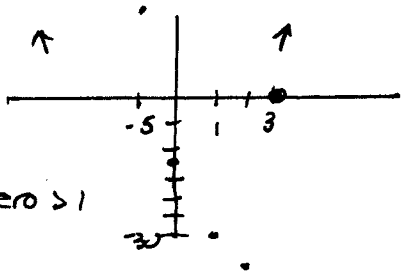
(a) *Before* finding the zeros of f(x), list all POSSIBLE rational roots $\pm\{1, 2, 3, 4, 6, 12, \frac{1}{2}, \frac{3}{2}\}$

(b) Find the zeros. Show all steps clearly including trying numbers that did not work.

(c) Factor P completely.

Use graph for strategy *

$$\begin{array}{r|rrrrr} 1 & 2 & -5 & 5 & -20 & -12 \\ & & 2 & -3 & 2 & -18 \\ \hline & 2 & -3 & 2 & -18 & -30 \end{array}$$



* 1 doesn't work but there must be a zero > 1

$$\begin{array}{r|rrrrr} 2 & 2 & -5 & 5 & -20 & -12 \\ & & 4 & -2 & 6 & -28 \\ \hline & 2 & -1 & 3 & -14 & -40 \end{array}$$

* must be a zero > 2

$$\begin{array}{r|rrrrr} 3 & 2 & -5 & 5 & -20 & -12 \\ & & 6 & 3 & 24 & 12 \\ \hline & 2 & 1 & 8 & 4 & 0 \end{array}$$

$x = 3$ is a zero, $x - 3$ is a factor

* so far $P(x) = (x-3)(2x^3 + x^2 + 8x + 4)$ you can actually factor by grouping here, but I continued finding zero process for practice

New list of possible rational zeros $\left\{ \begin{matrix} \cancel{1}, \cancel{2}, 4, \frac{1}{2} \\ -1, 2, 4, -\frac{1}{2} \end{matrix} \right\}$ * Exclude $x = 1, 2$ since already showed they don't work

* Looking at evolving graph, there must be a zero for $x < 0$ Also, Descartes' rule (optional) tells us there are no more + real roots

(*) Using depressed equation now!!

$$\begin{array}{r|rrrr} -1 & 2 & 1 & 8 & 4 \\ & & -2 & 1 & -9 \\ \hline & 2 & -1 & 9 & -5 \end{array}$$

* -1 doesn't work. We can not say $(-1, -5)$ is on graph since we are now working on depressed polynomial. We can find $P(-1) = 20$ so there must be a zero between $x = -1$ and $x = 0$ (from graph)

$$\begin{array}{r|rrrr} -\frac{1}{2} & 2 & 1 & 8 & 4 \\ & & -1 & 0 & -4 \\ \hline & 2 & 0 & 8 & 0 \end{array}$$

$x = -\frac{1}{2}$ is a zero

$x + \frac{1}{2}$ is a factor

So $P(x) = (x-3)(x+\frac{1}{2})(2x^2+8)$, zeros are $\{3, -\frac{1}{2}, \pm 2i\}$

remaining zeros $\left\{ \begin{aligned} 2x^2 + 8 &= 0 \\ x^2 &= -4 \\ x &= \pm 2i \end{aligned} \right.$

factors are: $(x-3)(x+\frac{1}{2})(x-2i)(x+2i)$

also accept $(x-3)(x+\frac{1}{2})(2x^2+8)$

(10) Suppose the revenue, in dollars from sales of a product is a function of the unit price, in dollars that is charged. If the revenue is given by the function

$R(p) = -\frac{1}{2}p^2 + 300p$ max at vertex $p = \frac{-b}{2a} = \frac{-300}{2(-1/2)} = 300$ $R(300) = 45{,}000$

a) What is the maximum revenue? \$ 45,000

b) What price should be charged to achieve maximum revenue? \$ 300 per unit

*Answer using appropriate units.

(6 points)

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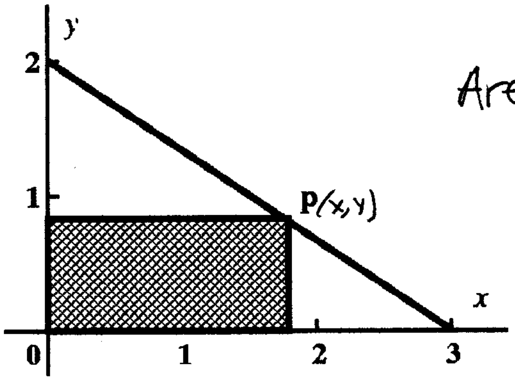
(11) A point P(x,y) lies in the first quadrant on the graph of the line $y = 2 - \frac{2}{3}x$. From the point P, perpendiculars are drawn to both the x-axis and the y-axis. What are the dimensions of the rectangle of larges area thus formed?

(Note. Don't be mislead by the picture. P is not a fixed point, this is just one possible location for it)

(10 points)



Area = $lw = xy = x(2 - \frac{2}{3}x)$

$A(x) = 2x - \frac{2}{3}x^2$

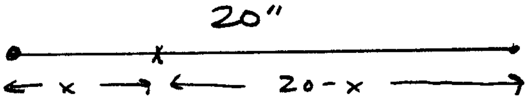
max at vertex $x = \frac{-b}{2a} = \frac{-2}{2(-\frac{2}{3})} = \frac{2}{4/3} = \frac{3}{2}$

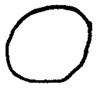
Dimensions

length $= x = \frac{3}{2}$

ht $= y = 2 - \frac{2}{3}x = 1$

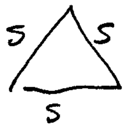
(12) A piece of wire 20 inches long is cut into two pieces. The first is bent into a circle, the second is bent into an equilateral triangle. How should the wire be cut in order to minimize the total enclosed area. (10 points)





$2\pi r = x$

$r = \frac{x}{2\pi}$

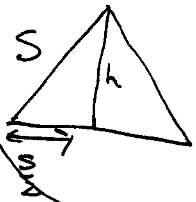


$3s = 20 - x$

$s = \frac{20-x}{3}$

$$\text{Area} = \pi r^2 = \pi\left(\frac{x}{2\pi}\right)^2 = \frac{x^2}{4\pi}$$

$$\text{Area} = \frac{1}{2}bh = \frac{1}{2}sh = \frac{1}{2}\left(\frac{20-x}{3}\right)\frac{\sqrt{3}}{2}\left(\frac{20-x}{3}\right) = \frac{\sqrt{3}}{36}(20-x)^2$$



$h^2 + \left(\frac{s}{2}\right)^2 = s^2$

$h^2 = s^2 - \frac{s^2}{4} = \frac{3s^2}{4}$

$h = \frac{s\sqrt{3}}{2} = \frac{\sqrt{3}}{2}\,\frac{20-x}{3}$

$$A(x) = \frac{1}{4\pi}x^2 + \frac{\sqrt{3}}{36}(20-x)^2$$

$$= \frac{1}{4\pi}x^2 + \frac{\sqrt{3}}{36}(400 - 40x + x^2)$$

$$= \frac{1}{4\pi}x^2 + \frac{400\sqrt{3}}{36} - \frac{40\sqrt{3}}{36}x + \frac{\sqrt{3}}{36}x^2$$

$$A(x) = \left(\frac{1}{4\pi} + \frac{\sqrt{3}}{36}\right)x^2 - \frac{10\sqrt{3}}{9}x + \frac{100\sqrt{3}}{9}$$

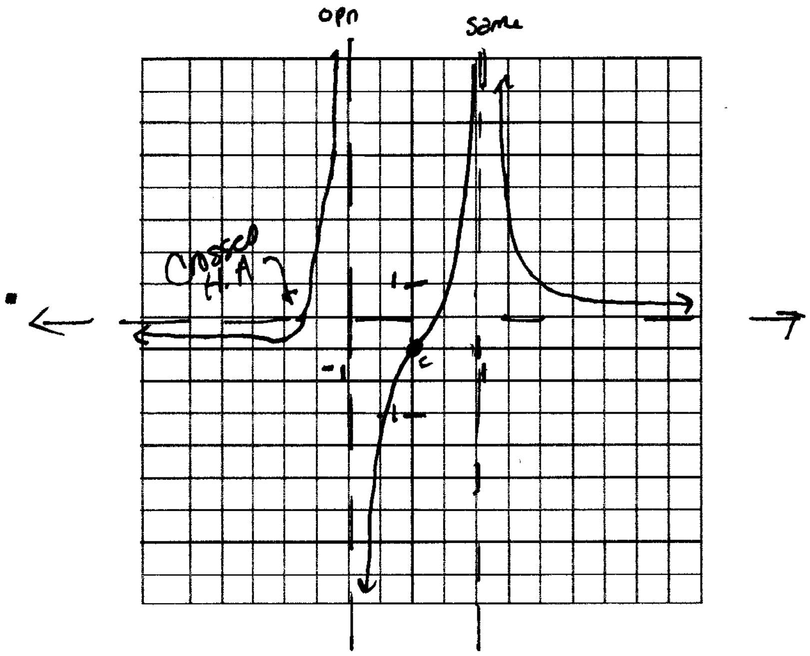
Min occurs at vertex

simplify complex fraction

$$x = \frac{\frac{10\sqrt{3}}{9}}{2\left(\frac{1}{4\pi} + \frac{\sqrt{3}}{36}\right)} = \frac{\frac{10\sqrt{3}}{9}}{\frac{1}{2\pi} + \frac{\sqrt{3}}{18}} \cdot \frac{18\pi}{18\pi} = \frac{20\pi\sqrt{3}}{9 + \sqrt{3}\,\pi}''$$ to make circle

(13) Sketch the graph of $y = \dfrac{x^3}{2(x-1)^2(x+1)}$ Show any asymptotes. Be sure to sure all work including discussion of asymptotes, intercepts, and behavior. Show scale.

(12 points)



domain : $x \neq \pm 1$

V.A : $x = 1$ $x = -1$ ← give as equation

approach: same opp

H.A. : $y = \frac{1}{2}$

Cross? : $\frac{1}{2} = \frac{x^3}{2(x-1)^2(x+1)}$

$x^3 = (x-1)^2(x+1)$

$x^3 = (x^2-2x+1)(x+1)$

$x^3 = x^3 + x^2 - 2x^2 - 2x + x + 1$

$0 = -x^2 - x + 1$

$0 = x^2 + x - 1$

$x = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2} \approx \begin{matrix} .62 \\ -1.6 \end{matrix}$

X-int : $x = 0$ cross

y-int $y = 0$

(14) Given the polynomial $f(x) = -4x^3 + 4x^2 + 7x + 2$ (12 points)

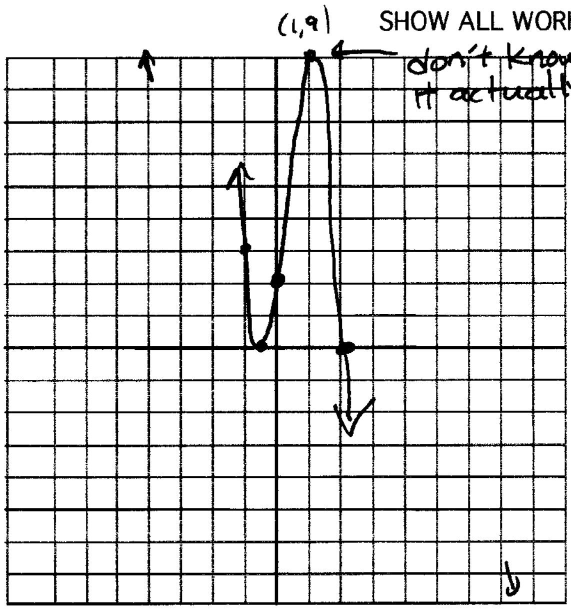
(a) discuss end behavior As $x \to \infty$, $f(x) \to -\infty$; $x \to -\infty$, $f(x) \to \infty$

(b) find the y intercept $(0, 2)$

(c) find the x intercepts and discuss the behavior near them.

(d) plot one additional point for accuracy and sketch the graph.

SHOW ALL WORK



show whole process

Possible rational zeros $\pm\{1, 2, \frac{1}{2}, \frac{1}{4}\}$

$$\begin{array}{r|rrrr} 1 & -4 & 4 & 7 & 2 \\ & & -4 & 0 & 7 \\ \hline & -4 & 0 & 7 & 9 \end{array}$$

1 doesn't work but $f(1) = 9$

$$\begin{array}{r|rrrr} 2 & -4 & 4 & 7 & 2 \\ & & -8 & -8 & -2 \\ \hline & -4 & -4 & -1 & 0 \end{array}$$

2 works

$$f(x) = (x-2)(-4x^2 - 4x - 1)$$
$$= -(x-2)(4x^2 + 4x + 1)$$
$$= -(x-2)(2x+1)^2$$

x-int.	2	$-\frac{1}{2}$
behavior:	cross	bounce

(15) For each of the following angles, determine which quadrant it is and find the reference angle. Answer should be in the units given. 1 point each blank.

ANGLE	QUADRANT	REFERENCE ANGLE
220°	3	40°
100°	2	80°
92°	2	88°
-300°	~~3~~ 1	60°
$8\pi/7$	3	$\frac{\pi}{7}$
$5\pi/3$	4	$\frac{\pi}{3}$
$11\pi/10$	~~2~~ 3	$\pi/10$

(16) For each of the following, find 4 angles, one in each quadrant, having the given angle as a reference angle. Answer in the units given. 1 point each blank.

12°	12°	168°	192°	348°
45°	45°	135°	225°	315°
$\pi/10$	$\frac{\pi}{10}$	$\frac{9\pi}{10}$	$\frac{11\pi}{10}$	$\frac{19\pi}{10}$
$2\pi/5$	$\frac{2\pi}{5}$	$\frac{3\pi}{5}$	$\frac{7\pi}{5}$	$\frac{8\pi}{5}$
1	1	$\pi - 1$	$\pi + 1$	$2\pi - 1$